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# A Model for Target Tracking in the Presence of Geographic Constraints

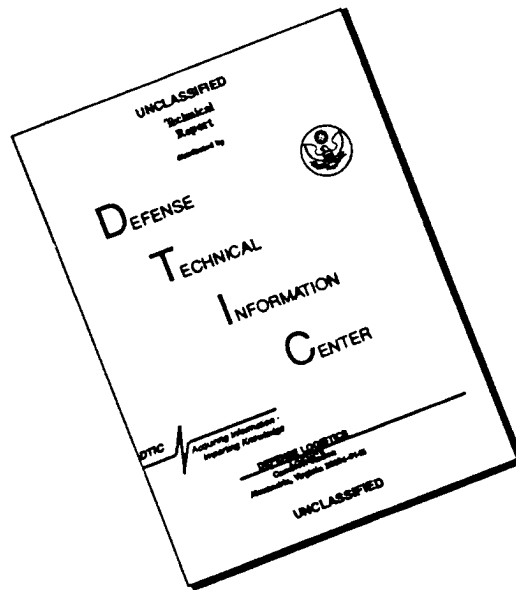
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## **PREFACE**

This report was prepared under Project No. 33CWF55, "Submarine Contact Management Subtask of the NUWC Division Newport Submarine Combat Control Task," principal investigator A. Silva (Code 2211). The sponsoring activity is the Office of Naval Research, program manager J. Fein (ONR-333).

The technical reviewer for this report was R. L. Streit (Code 2214).

**Reviewed and Approved: 2 January 1996**

A handwritten signature in dark ink, appearing to read 'P. A. La Brecque', with a stylized flourish at the end.

**P. A. La Brecque**  
**Head, Combat Systems Department**

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## TABLE OF CONTENTS

	Page
LIST OF ILLUSTRATIONS .....	ii
LIST OF TABLES .....	ii
INTRODUCTION .....	1
SYSTEM CONSIDERATIONS .....	2
Point Estimate Constraints .....	2
Constrained Uncertainty Regions .....	3
A NON-HOMOGENEOUS PROCESS NOISE MODEL FOR TRACKING NEAR LANDMASS CONSTRAINTS.....	4
SIMULATION RESULTS .....	10
CONCLUSIONS.....	17
REFERENCES.....	18

## LIST OF ILLUSTRATIONS

Figure		Page
1a	Perspective Plot of the Mixture Representation of Land for the Triangular Island Parameterized in Table 1 .....	7
1b	Shoreline Contour Surrounding the 36 Component Ellipses of the Mixture Representation for the Triangular Island Parameterized in Table 1 .....	7
2	Samples of the Process Noise Field Showing Mean Vectors and Covariance Ellipses as a Function of Location .....	8
3	Perspective Plot of the Window Function $C_0(r)$ .....	10
4a	Data-Rich Scenario .....	11
4b	Algorithm Output When No Land Constraint Is Applied for Data-Rich Scenario.....	12
4c	Algorithm Output Using Non-Homogeneous Process Noise Model for Data-Rich Scenario .....	12
5a	Sparse-Data Scenario .....	14
5b	Algorithm Output When No Land Constraint Is Applied for Sparse-Data Scenario .....	14
5c	Algorithm Output Using Non-Homogeneous Process Noise Model for Sparse-Data Scenario .....	15
6a	Algorithm Output When No Land Constraint Is Applied for No-Data Scenario .....	16
6b	Algorithm Output Using Non-Homogeneous Process Noise Model for No-Data Scenario .....	16

## LIST OF TABLES

Table		Page
1	Simulation Parameters for Land Parameters and Process Parameters .....	9
2	Simulation Parameters for Target Process and Measurement Process .....	13

# **A MODEL FOR TARGET TRACKING IN THE PRESENCE OF GEOGRAPHIC CONSTRAINTS**

## **INTRODUCTION**

Incorporating geographic constraints and environmental information into the target tracking process is a nontrivial problem. To obtain accurate target estimates, accurate signal modeling and pertinent state constraints must both be present in a fully integrated algorithm. Preconditioning data or applying state constraints after estimation does not yield mathematically optimal state estimates for the typical problems encountered.

Operations in littoral waters demand the ability to accurately track contacts in the presence of landmass constraints. Minefields, under appropriate conditions, can also strongly bias the probability of a target presence. Weather, sea state, and visibility can constrain or severely limit target velocity and maneuverability. Because the Navy's submarine focus has been on open-ocean operations and antisubmarine warfare (ASW), these environment-related tracking issues have received little attention. While some Navy tracking products such as the Composite Area Analysis Module (CAAM), the ASW Tactical Decision Aid (ASWTDA), and Visual Integration of Scene Information Over Networks (VISION) allow for the storage, retrieval, and display of environmental information (e.g., charting, bathymetry, sound velocity profiling, shipping noise statistics) (references 1-3), current target motion analysis algorithms generally do not utilize environmental information. In some instances, multiple hypothesis techniques are employed to make a propagation path decision when the true path is unknown; yet, multipath effects and the vagaries of sound propagation in shallow water remain poorly integrated into the contact management function.

As an initial step toward fully incorporating environmental information into target motion analysis, this report formulates a non-homogeneous process noise model for application to target tracking near landmasses. Some of the principal considerations in developing such a model are discussed, and the results of scaled simulations are presented to illustrate the model's effect on tracking algorithm performance under varying data conditions.

## SYSTEM CONSIDERATIONS

### POINT ESTIMATE CONSTRAINTS

A fully coupled, fully integrated target tracking algorithm is most desirable, as one is often faced with integrating target solutions from diverse and independent sources. Even when embedded in solutions presented by sensor data processing, kinematic constraints induced by the environment must be considered when integrating solutions that may be time-late, aged by prediction, or of uncertain target association. Inclusion of environmental constraint models becomes even more crucial when a predictive capacity is required, such as when searching for a lost contact, or when performing *in-situ* tactical planning. Current tracking algorithms—if they consider environmental constraint information at all—typically treat such information as hard limits on traditional point estimates of target state, with little regard for the ramifications for the state probability distribution.

One straightforward method for constraining the point estimate in an iterative algorithm is simply to modify the magnitude of the state update in each iteration to ensure constraint adherence. This approach is perhaps the least desirable means of constraint implementation since the state update prediction does not incorporate the constraint, and convergence to a suboptimal solution (non-extrema of the constrained performance index) is possible. In addition, the uncertainty estimate is not affected by this method of constraint implementation; in fact, when a constraint is active (the estimate lies on the constraint boundary), a large portion of the solution uncertainty region may violate the constraint. Moreover, constraining point estimates in this way can lead to significant prediction errors.

A more rigorous method utilizes penalty functions and Lagrange multipliers (reference 4). With this method the constraint is incorporated in the state update calculation, and under appropriate conditions an optimal solution (an extrema of the corresponding cost function) is ensured. When constraint conditions are active, the level curves of the modified cost function may provide qualitative indications of the solution uncertainty region. However, since such methods are developed for functional minimization/maximization, they provide no direct means of quantitative uncertainty characterization. Although the cost function may give an indication of the corresponding uncertainty region, there remains the problem of characterizing point estimates near (but not on) the constraint boundary. Here, the inactive constraints will not be incorporated in the cost function and thus not properly accounted for in the uncertainty display. As a result, parts of the uncertainty region may violate the constraint and, when constraints become active, abrupt changes can occur in the cost function and corresponding solution uncertainty assessment. Moreover, the additional processing required to properly handle the Lagrange multipliers at each iteration is computationally nontrivial. Unconstrained penalty function methods approximate the constrained functional minimization/maximization problem by a related unconstrained problem. Although the discontinuity at the constraint boundary is not



a problem in this case, the lack of quantitative uncertainty measures and the lack of appropriate target motion models remain as issues.

## CONSTRAINED UNCERTAINTY REGIONS

In the presence of constraints, full target estimate characterization requires the treatment of non-Gaussian target uncertainty descriptions. A constrained target motion model must utilize the non-homogeneous state transition probabilities and nonstationary errors that characterize target motion near constraint boundaries. Rigorous modeling typically requires numerically intensive techniques.

An analog of the limited state update is the renormalized, windowed, probability density function (PDF) uncertainty description. Here, the target PDF is developed in the absence of any kinematic constraints and, typically, Gaussian assumptions/approximations are used. This PDF is then multiplied by a window function that is unity over regions of navigable water and zero where constraints are violated (i.e., over land). The result must then be renormalized to account for the lost probability mass in the inadmissible region. Such a representation of target state probability will not violate the constraint but may not be representative of reality either, especially when searching for a lost target near a peninsula. The combination of a point estimate constraint and a windowed PDF is a crude but effective model for the constrained state estimation process.

Because of the nonlinear, non-Gaussian nature of the problem, theoretically rigorous modeling is generally achieved by using numerical techniques. Grid-based methods maintain numerical values of the target state probability density in an array representing a grid of points in the state space. The equations for predicting a target location or incorporating new measurement information are not solved in closed form. Instead, the PDF is simply evaluated at each grid vertex based on past conditional probability density descriptions evaluated on similar grids. A point solution is extracted from the grid based on the chosen metric, or the solution is interpolated using the surrounding grid vertices. Marginalization is achieved through summation over the appropriate indices, and uncertainty regions are determined by level curves (contour maps) of the PDF on the grid. The difficulties with such methods are their sensitivity to grid resolution and their computational burden. Adaptive adjustment of the grid location and resolution is helpful but carries a computational burden of its own. If a significant percentage of the density lies between adjacent grid vertices, or beyond grid boundaries, information will be lost, and information lost due to a poorly maintained grid is often difficult to retrieve. An adaptive, grid-based method is employed in the NODESTAR algorithm developed by Metron Inc. (reference 5).

Monte Carlo methods avoid the need to manage a grid. Instead, the target state PDF is represented in histogram fashion by a large number of points ("estimates") in the state space. These points may be thought of as the states of notional targets, and each of these notional targets evolves according to a statistical process model that accounts for the constraint (landmass) presence. The probability distributions governing the choice of target velocity for

propagation in time can be adapted to include a geographic-constraint-avoidance model. Such methods are not particularly suited to incorporating new information but are adept at predicting a target PDF from a given reference time to some future time, in light of the given constraints. As with grid-based methods, there is a significant computational burden associated with Monte Carlo techniques because of the large number of data points required to generate a histogram representing the target distribution. A Monte Carlo tracker applied to the landmass avoidance problem is currently under investigation at the Naval Surface Warfare Center (reference 6).

Implicit in both the grid and Monte Carlo methods are statistical descriptions of target motion in the proximity of the land boundary. Should the predicted position violate the landmass constraint during the next prediction interval, remedial action is taken. In the case of the Monte Carlo method of reference 6, notional targets that violate the constraint are replaced with samples that do not violate the constraint. The actual characterization of the velocity distribution from which samples are drawn can be tailored to the operating conditions and the expected target action. Some of the motions emulated in reference 6 include fleeing target, random walk, front motion, intercept, shadow, and evasive maneuver. In the grid-based method of reference 5, the likelihood of signal detection (which is zero for target positions over land) automatically zeros the target probability density in the constraint region. The velocity distribution is also maintained on a grid, and its evolution is driven by the measurement and detection likelihood.

### **A NON-HOMOGENEOUS PROCESS NOISE MODEL FOR TRACKING NEAR LANDMASS CONSTRAINTS**

In applying modern control and estimation theory to the tracking of ocean targets, the following constant-velocity motion model is generally applied:

$$x_{k+1} = F_k x_k + G_k \omega_k, \quad (1)$$

$$z_k = h_k(x_k) + \eta_k, \quad (2)$$

where  $x_k$  is the vector representing position and velocity in Cartesian coordinates at time  $t_k$ , and  $z_k$  is the corresponding measurement vector. Here, the plant matrix  $F_k$  defines the deterministic component of the target state transition for time index  $k$  to  $k + 1$ , and it is solely a function of the time increment ( $t_{k+1} - t_k$ ). The process noise term  $\omega_k$  accounts for random errors and stochastic disturbances (maneuvers) in the target motion description, and  $G_k$  is usually considered a constant matrix. The measurements  $z_k$  can be nonlinearly related to the target state (e.g., bearing and range), although in the simulations that follow in this report direct observations of target position in Cartesian coordinates were used. Both the measurement noise  $\eta$  and process noise  $\omega$  are typically considered to be zero-mean Gaussian random processes with fixed covariance matrices  $R$  and  $Q$ , respectively.

Classic applications of Kalman filter theory for target tracking employ homogeneous, stationary, Gaussian, white noise sequences. In contrast, the method to be described here uses

non-homogeneous process noise and is currently being investigated at the Naval Undersea Warfare Center (NUWC) Division, Newport, RI, for use when tracking in the presence of landmass constraints. Traditionally, a mean noise value is lumped into the deterministic model, either as a control input or in the state transition function. In the development that follows, it will be instructive to consider process noise models with nonzero mean.

It is assumed here that the waterborne targets of interest are not able to traverse land barriers. In addition, it is assumed that when these targets are in the vicinity of land they have a greater tendency to maneuver (to avoid running aground). The first assumption can be approximately modeled in the target motion equations by a large coefficient of friction over the landmass regions. The effects of a frictional force can be included in the state transition function or, as will be seen, as a mean on the process noise that is parametrically described in terms of the current target velocity. The tendency for the target to maneuver can be modeled by an elevated value of the process noise variance. As a result, the desired target motion model in the vicinity of land can be approximated using state dependent (non-homogeneous) process noise. The problem remains, then, to characterize this state dependence in a manner that provides a smooth transition from open ocean to near-land conditions. This characterization is achieved in a simple and effective manner by using a Gaussian sum representation of the landmass regions. (Streit and Luginbuhl (reference 7) applied Gaussian sum representations to the multitarget/multisensor data association problem, and their work inspired such application to the present problem.)

Because the primary reason for using process noise in this instance is to model the velocity bias and maneuver uncertainty, in the discussion that follows the process noise is considered to be concentrated in velocity. The impact on position is achieved through the integration implicit in the state transition function. For the application at hand, one can consider the process noise to arise from multiple sources, which will be termed *maneuver influence sources*. The relative contribution of each of these sources depends on the target location. One maneuver influence source will be defined for the open ocean and, since it will be small relative to the other sources and negligible in their presence, it will be considered to have constant weight throughout the region of interest. In addition to the open-ocean source, there is a maneuver influence source associated with each of the components in a mixture representation of land. These sources receive a weight that is proportional to a target's proximity to the particular landmass component in question; i.e., these sources are weighted by an exponential form directly analogous to a Gaussian probability density with mean and covariance determined by that particular component of the landmass representation. Finally, these weights are normalized so that the sum over all sources is unity; see equation (3). For simplicity and tractability, each of these process noise sources is considered to be Gaussian, with specified mean and covariance. The open-ocean source is modeled as zero mean ( $\mu_0 = 0$ ) and with a relatively small covariance typical of that used in classic application of Kalman filter equations to the constant-velocity target tracking problem. The sources associated with landmass components are modeled with a mean value emulating a frictional force that prevents motion on land and a small reflective velocity tailored to model any land avoidance tendency. The covariance of these land sources is relatively high, indicative of a strong tendency to maneuver. Thus, for land mode  $i$  ( $i = 1, 2, \dots, n$ ), the covariance is fixed at  $Q_i$  and the mean is given by

$$\mu_i(v) = -\alpha v + v_i, \quad (3)$$

where  $v$  is the target velocity vector, the form of  $v_i$  is designed to model whatever land-avoidance action is expected of the target, and  $\alpha$  corresponds to the coefficient of friction. In the simulation results that follow,  $v_i$  is simply a small velocity vector directed away from the land mode in question and toward the open ocean. It is important that the coefficient  $\alpha$  be close enough to unity that a target approaching land slows sufficiently in relatively few update intervals. In tests performed to date, the value  $\alpha = 1$  has been used.

The weighted sum of Gaussian random variables is also Gaussian distributed with a mean value given by the weighted sum of the individual means and a covariance given by the weighted sum of the individual covariances.

Putting all of the foregoing elements together gives the following functions for the mean and covariance of the Gaussian process noise model:

$$\omega_k \approx N[\mu(\hat{x}_{tk}), Q(\hat{x}_{tk})], \quad (4)$$

$$\mu(x) = \sum_{i=0}^n C_i(r) \mu_i(v), \quad (5)$$

$$Q(x) = \sum_{i=0}^n C_i^2(r) Q_i, \quad (6)$$

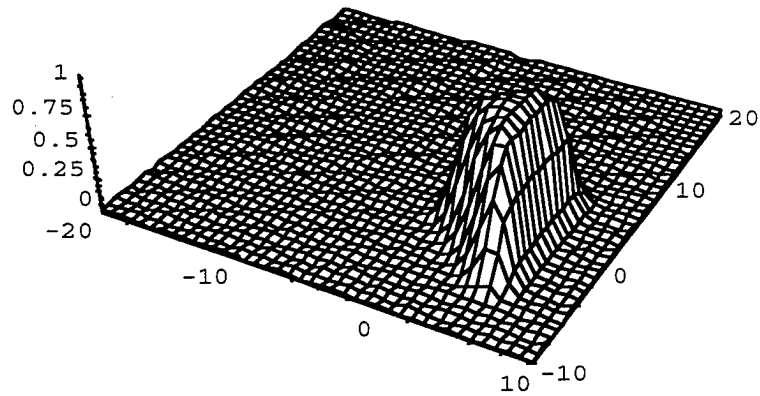
where  $\hat{x}_{tk} = (\hat{r}_{tk}, \hat{v}_{tk})$  is the estimated target state (position and velocity) at time  $t_k$ , the  $\mu_i$  are defined in equation (3), and coefficients  $C_i(r)$  are defined as follows:

$$C_0(r) = \frac{\pi_0}{\pi_0 + \sum_{j=1}^n \pi_j N(m_j, \Sigma_j)|_r}, \quad (7a)$$

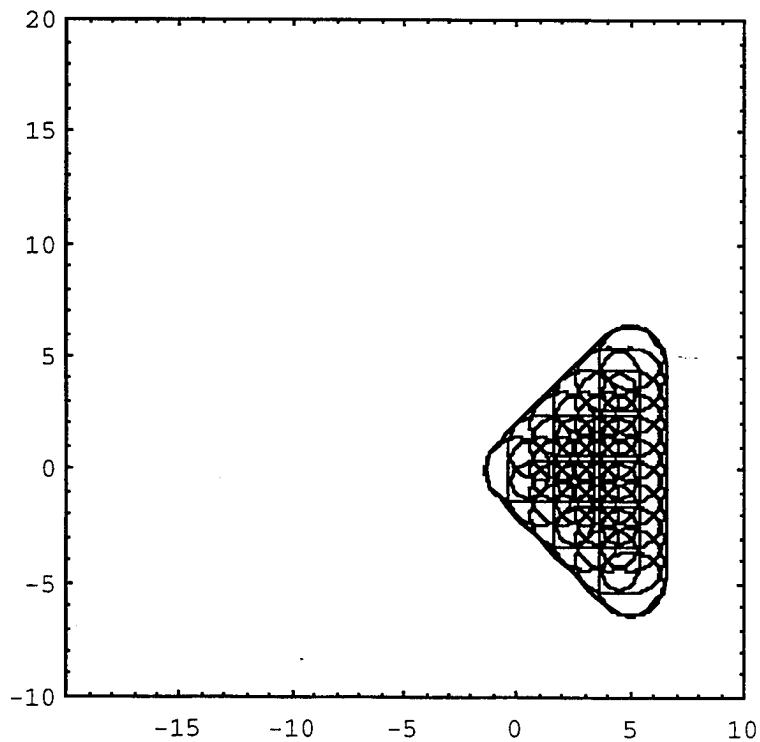
$$C_i(r) = \frac{\pi_i N(m_i, \Sigma_i)|_r}{\pi_0 + \sum_{j=1}^n \pi_j N(m_j, \Sigma_j)|_r}, \quad i = 1, 2, \dots, n. \quad (7b)$$

Here,  $N(m_i, \Sigma_i)|_r$  signifies the multivariate Gaussian density function, with mean  $m_i$  and covariance matrix  $\Sigma_i$  evaluated at the point  $r$ . It is sufficient, for the purposes of this formulation, to equate  $\pi_i$  to unity for  $i > 0$ , i.e.,  $\pi_1 = \pi_2 = \dots = \pi_n = 1$ . The sum appearing in the denominator of equation (7),

$$\mathcal{L}(r) = \sum_{j=1}^n \pi_j N(m_j, \Sigma_j)|_r, \quad (8)$$



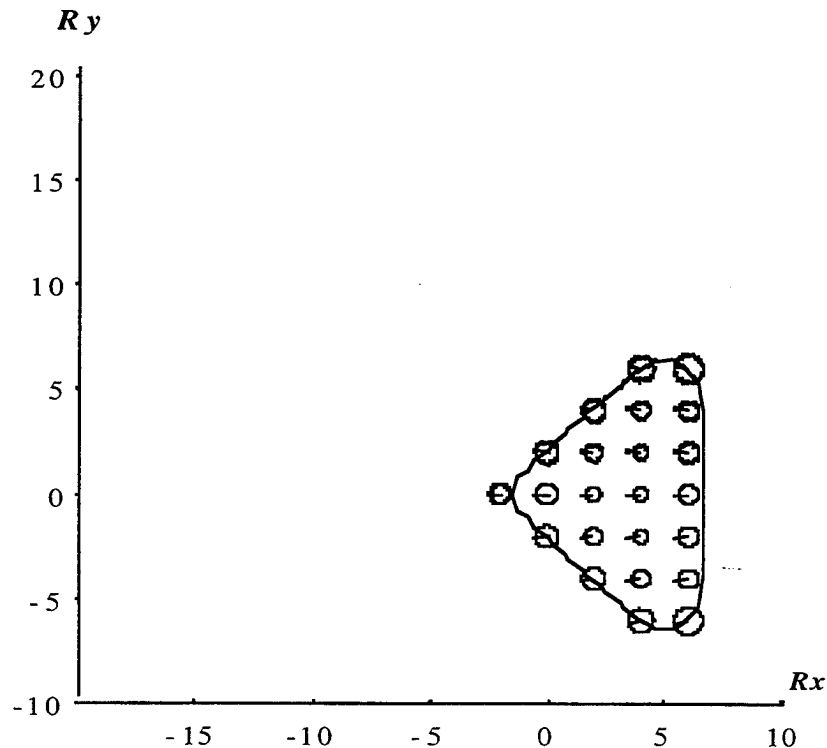
**Figure 1a. Perspective Plot of the Mixture Representation of Land for the Triangular Island Parameterized in Table 1**



**Figure 1b. Shoreline Contour Surrounding the 36 Component Ellipses of the Mixture Representation for the Triangular Island Parameterized in Table 1**

corresponds to the mixture representation of the landmass region (see figure 1). The contour parametrically defined by  $\ell(r) = \gamma$  is designed to approximate the shoreline in the region of interest (figure 1b); the values of  $(m_i, \Sigma_i, i = 1, 2, \dots, n)$  and  $n$  are design parameters to be specified for each region of interest. The values of  $\pi_0$  and  $\gamma$  are design parameters that are expected to be fixed for all regions. An illustration of the process noise field as a function of location is shown in figure 2 for the land boundary shown in figure 1 and a hypothesized target moving in the positive  $x$ -direction.

The descriptive parameters for figure 2 are given in table 1. The figure shows a grid sampling of the process noise mean vector field as a function of location; centered at the base of each vector is the ellipse representing the corresponding process noise covariance matrix. Over navigable water, the process noise mean and variance are approximately zero; they become significant only near land, where the maneuver influence sources are appreciable.



**Figure 2. Samples of the Process Noise Field Showing Mean Vectors and Covariance Ellipses as a Function of Location**

**Table 1. Simulation Parameters for Land Parameters and Process Parameters**

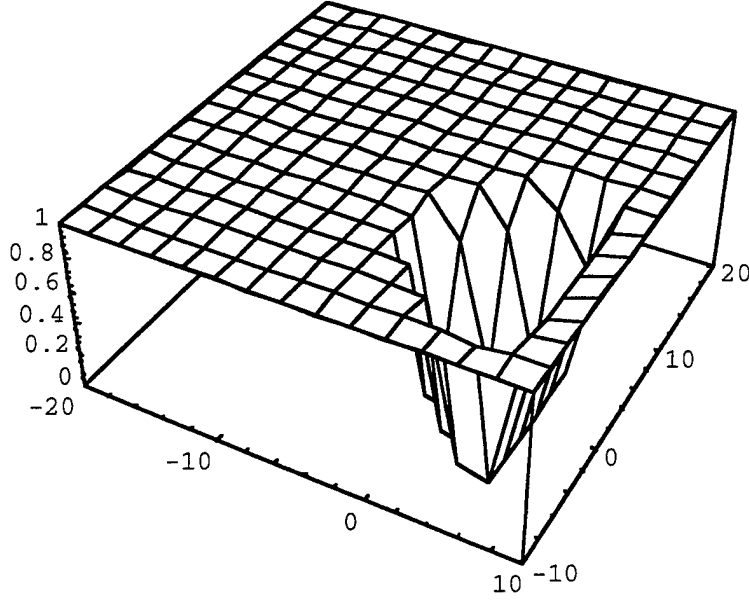
<u>Land Parameters:</u>	
$m_i = (k, j)$	
where $i = k(k+1) + j + 1$ , for $k = 0, 1, 2, 3, 4, 5$ ; $j = -i, \dots, 0, \dots, i$ ; i.e., $i = 1, 2, \dots, 36$ .	
$\Sigma_i = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$	$\gamma = 0.05$
<u>Process Parameters:</u>	
$v_i = [-0.1, \text{sign}(j)0.1]$ ,	
where $i = k(k+1) + j + 1$ , for $k = 0, 1, 2, 3, 4, 5$ ; $j = -i, \dots, 0, \dots, i$ ; i.e., $i = 1, 2, \dots, 36$ .	
$\mu_0 = (0, 0)$	$\pi_0 = 0.01$
$Q_0 = \begin{bmatrix} 10^{-8} & 0 \\ 0 & 10^{-8} \end{bmatrix}$	$Q_i = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$

Although the target kinematic model has been augmented to account for the presence of a landmass, the target density promoted by the Kalman tracking filter is still Gaussian in nature. The simplest method for including the land constraint in the target probability density is to employ a window function that is unity over water and zero over land. The function  $C_0(r)$  (equation 7a) is a smooth function that closely satisfies the above condition (see figure 3).

Thus, a simple approximation of the desired target density is provided by

$$p(x/z) \cong k C_0(r) N(\hat{x}, \hat{P})|_x, \quad (9)$$

where  $x^T = (r^T, v^T)$ ,  $k$  is a normalization constant such that  $p(x/z)$  integrates to 1, and  $\hat{x}, \hat{P}$  are the Kalman filter estimates of the target density mean and covariance, respectively. Note that if constraint regions are absent in the region of interest, this representation reverts to a simple Gaussian representation of the target density.



*Figure 3. Perspective Plot of the Window Function  $C_0(\mathbf{r})$*

## SIMULATION RESULTS

The formulation presented in the preceding section provides a compact representation of the constraint region in terms of a sum of Gaussian distributions, a smoothed window function for the target density, and a simple and effective means for implementing a non-homogeneous process noise model. Associated with each term of the Gaussian sum representation of land are corresponding process noise parameters  $\eta$  and  $Q$ . These parameters are most readily stored as velocity components of the land element state; i.e.,

$$\theta_i = \{m_i, v_i\}, \quad \Omega_i = \begin{bmatrix} \Sigma_i & 0 \\ 0 & Q_i \end{bmatrix}. \quad (10)$$

Thus, each land element has a data structure not unlike that of the target, with mean vector and covariance matrix; however, the information is used in a much different manner.

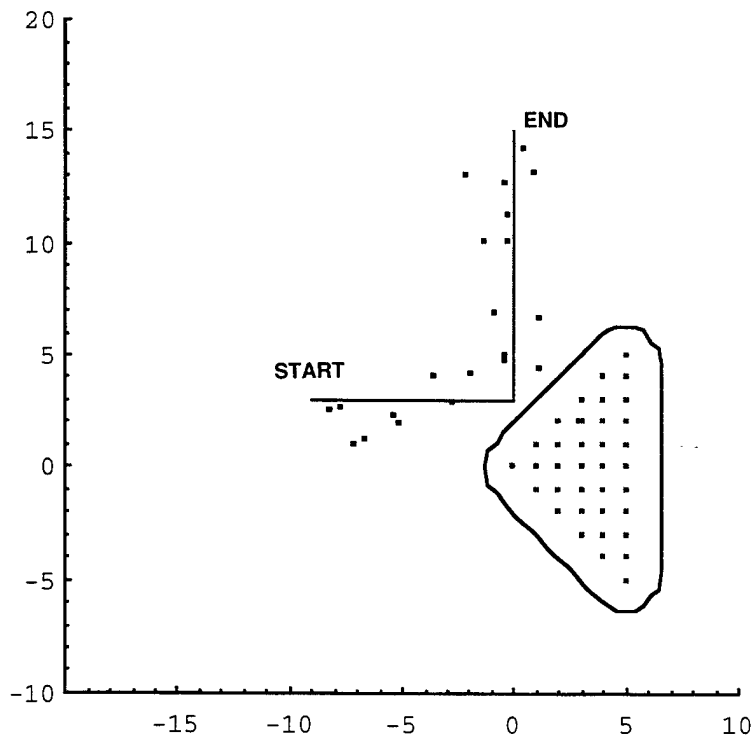
Several simulations were performed to illustrate the algorithm's performance under varying data conditions. Coordinates were normalized for convenient land mixture and target representation; time was similarly scaled to keep the simulation compact. A "data-rich" scenario was used to illustrate tracking through land-avoidance maneuvers; a "sparse-data" scenario was used to illustrate the track maintenance function; a "no-data" (loss of contact) scenario was used to illustrate the ability to predict the probability density. In all cases, the land model delineated by table 1 was used.



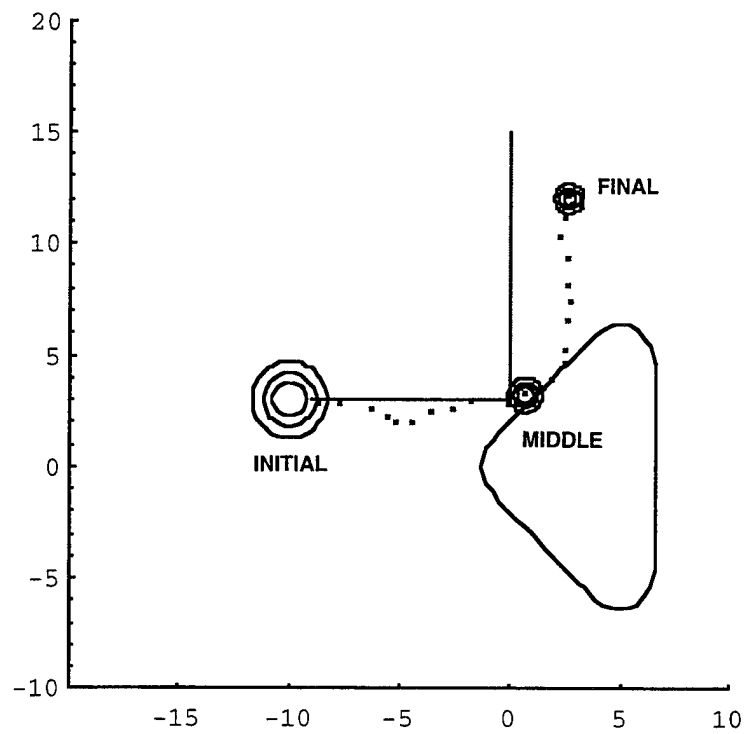
Figure 4 illustrates the simulation under data-rich conditions, where noisy measurements of position are available at each time index; i.e.,

$$z_k = H_k x_k + \eta_k, \quad (11)$$

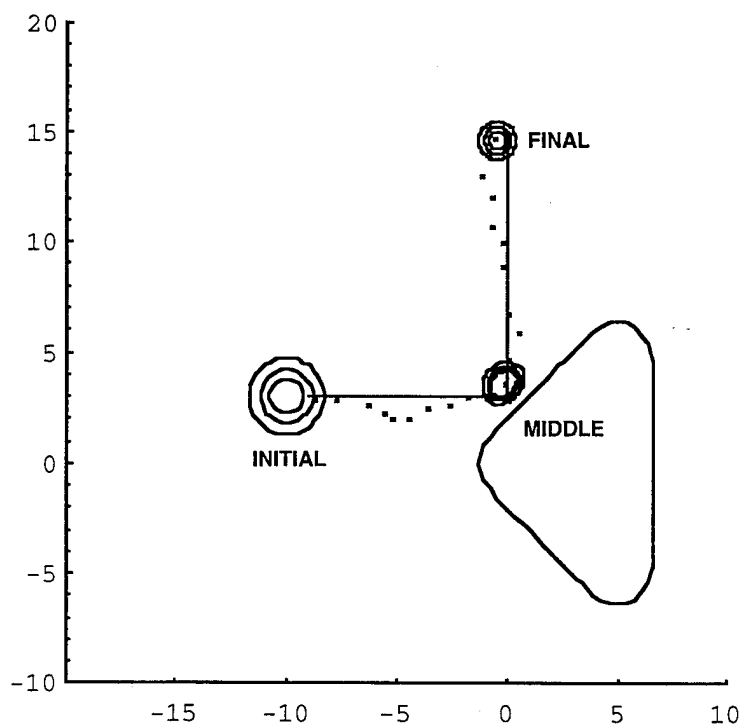
where  $\eta_k$  is the zero-mean Gaussian measurement noise at time  $t_k$ , and  $\eta_k \approx N(0, R_k)$ . In figure 4a the noisy measurements and simulated target track are superimposed on the plot of the 36 land mode mean positions and the resulting shoreline contour. The parameters pertaining to the target and measurement process are given in table 2. In all of the simulations presented here, the algorithm was initialized with the true target state at time zero ( $x_0$ ) and a moderate covariance ( $P_0$ ), since it was the behavior near the land boundary that was of primary interest. Figure 4b illustrates the algorithm's output (dotted curve) when the presence of land is not accounted for. Solution uncertainty contours are shown for the initial, middle, and final time solutions. Figure 4c shows the algorithm's output when the non-homogeneous process noise model of this report is applied. The increase in the algorithm's gain resulting from the land-induced process noise allows it to better track through the land-avoidance maneuver, yielding a more accurate solution and a more consistent solution uncertainty estimate.



*Figure 4a. Data-Rich Scenario*



**Figure 4b. Algorithm Output When No Land Constraint Is Applied for Data-Rich Scenario**



**Figure 4c. Algorithm Output Using Non-Homogeneous Process Noise Model for Data-Rich Scenario**

**Table 2. Simulation Parameters for Target Process and Measurement Process**

Target Process:

$$r_0^T = (-10, 3) \quad (\text{Initial target position})$$

$$v_0^T = (1, 0) \quad (\text{Initial target velocity})$$

$$v_{11}^T = (0, 1) \quad (\text{Instantaneous maneuver at } t = 11)$$

$$x_0^T = (r_0^T, v_0^T) \quad (\text{Position and velocity state vector})$$

$$dt = 1.0$$

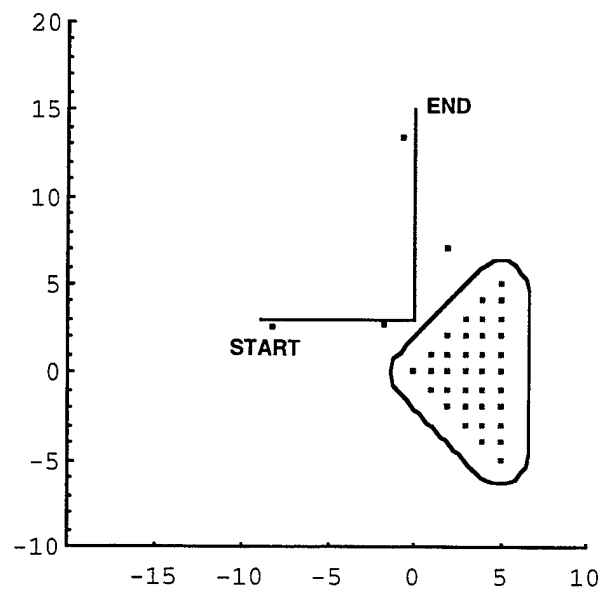
$$P_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 0.01 \end{bmatrix} \quad (\text{Initial target covariance})$$

$$F_k = \begin{bmatrix} 1 & 0 & dt & 0 \\ 0 & 1 & 0 & dt \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad G_k = \begin{bmatrix} dt & 0 \\ 0 & dt \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

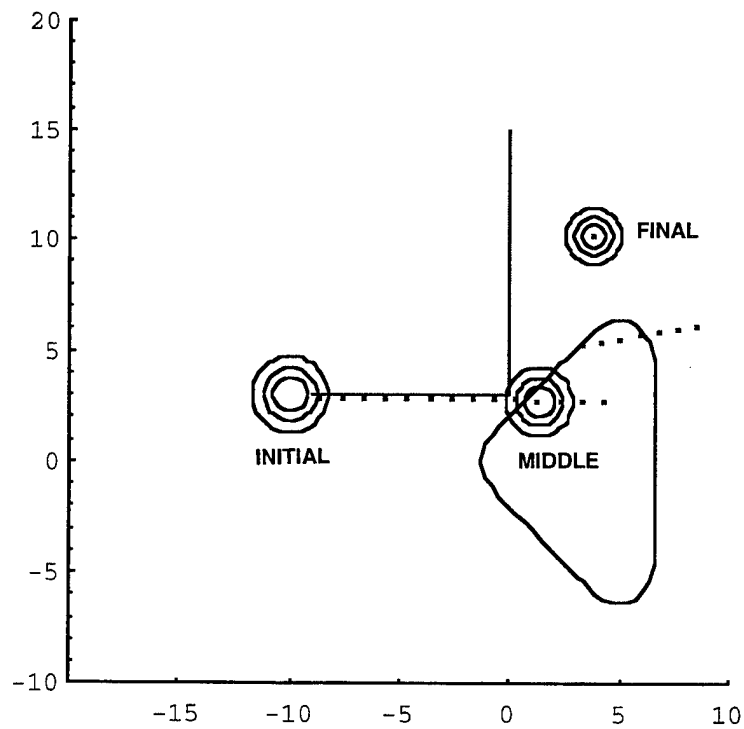
Measurement Process:

$$H_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad R_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

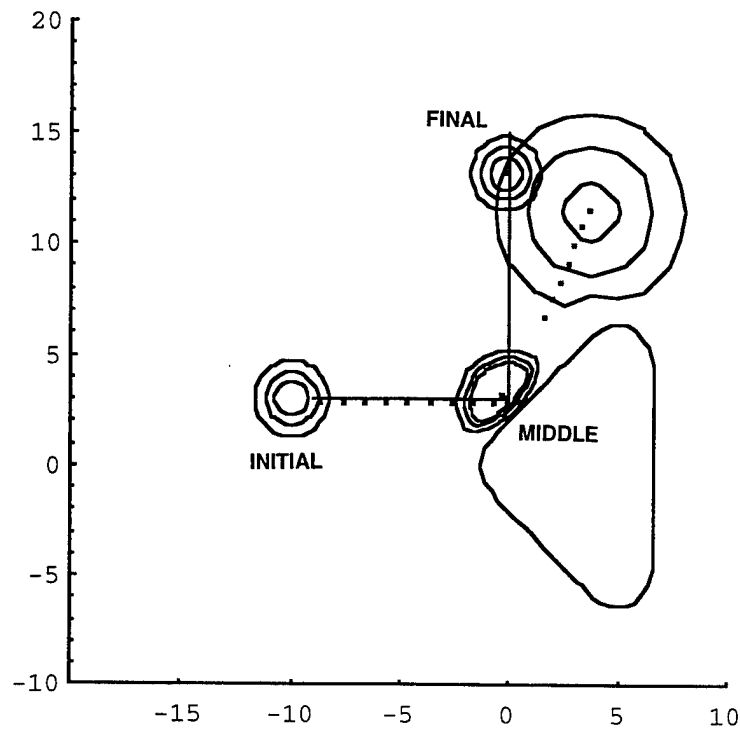
Figure 5 illustrates the same scenario under sparse-data conditions, where noisy position measurements are available every seventh time index, i.e., at times 1, 8, 15, and 22. Figure 5a shows the simulated target track and measurement set. Figure 5b shows the algorithm's output when land is not accounted for, and one can see an obvious violation of the land constraint. In figure 5c it is seen once again that the land-induced process noise increases the algorithm's gain to allow tracking through the land-avoidance maneuver, and it provides a consistent estimate of the solution uncertainty. The consistency of the uncertainty estimate is highlighted by including the uncertainty contours at time 21, just prior to processing the last position measurement.



*Figure 5a. Sparse-Data Scenario*

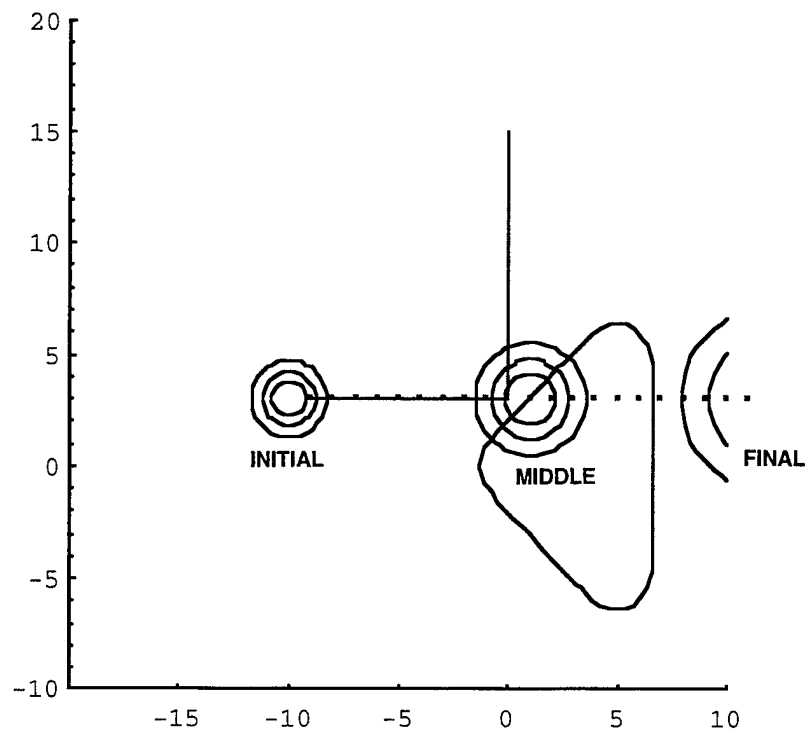


*Figure 5b. Algorithm Output When No Land Constraint Is Applied for Sparse-Data Scenario*

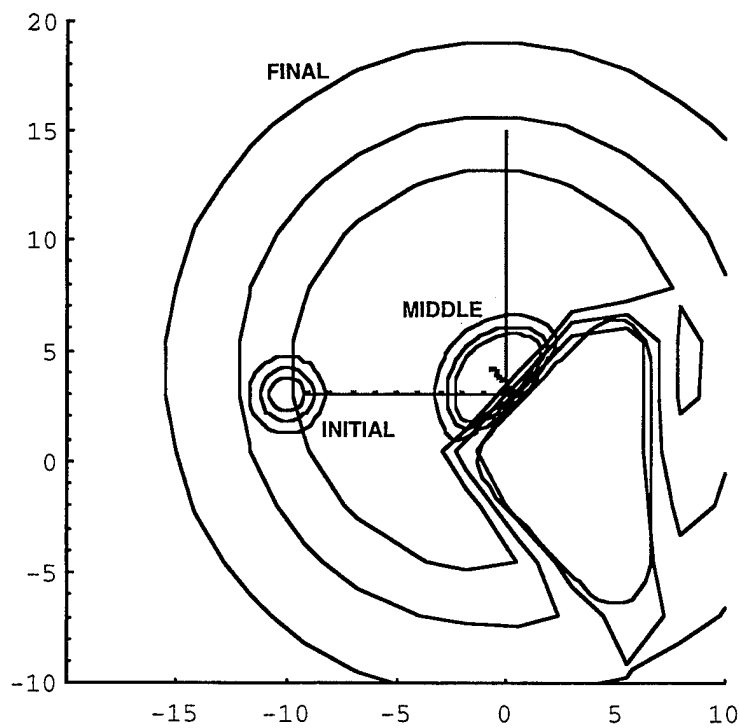


**Figure 5c. Algorithm Output Using Non-Homogeneous Process Noise Model for Sparse-Data Scenario**

The predictions for a lost contact (no data) are illustrated in figure 6 where the same scenario used in figures 4 and 5 is simulated without any available measurements. Because of the perfect initialization, the simulated and predicted tracks of figure 6a are coincident for the first half of the scenario. However, in the absence of a land-constraint model, the need for a land-avoidance maneuver is not accounted for in the last half of the predicted target track. Figure 6b illustrates how the non-homogeneous process noise field more accurately models the actual estimate uncertainty while also enforcing the land constraint on target motion. Note how the final time contours of the windowed density wrap around the landmass. Since the target could have chosen any land-avoidance maneuver—and none was more probable than another ( $v_i \approx 0$ )—little movement of the mean location is predicted after contact with shore. The predicted location uncertainty is, however, large and representative of the lack of maneuver information.



*Figure 6a. Algorithm Output When No Land Constraint Is Applied for No-Data Scenario*



*Figure 6b. Algorithm Output Using Non-Homogeneous Process Noise Model for No-Data Scenario*

## CONCLUSIONS

As has been demonstrated by the simulation results presented in this report, the net effect of applying an involved model of the target process noise field to the target tracking algorithm is that new data are readily incorporated in the manner of a Kalman filter, and the altered target behavior as it approaches a constraint boundary is accommodated in a compact form. Properly used, such a model will prevent constraint violation and will allow a more accurate portrayal of the state uncertainty near boundaries. The regions of constraint violation are represented using a mixture model that is fixed for a given region of interest, thus limiting the burden of excessive database interaction during filter iterations.

The simulation results illustrate the shift in focus from predictions of the mean estimate when in open water to predictions of the estimate uncertainty growth when near land. This refocus is a direct result of the assumption that an intelligent agent is directing the target platform and so it will maneuver to avoid running aground. The lack of specific maneuver information is manifested in the increase in the target estimate uncertainty. In the tracking model presented here, the lack of maneuver information is accounted for by using maneuver influence sources in the process noise formulation. The mode shift from the point estimate to the uncertainty region is an important aspect of tracking near constraint boundaries.

Although quantitative performance measures would be premature at this juncture, factors affecting performance include the number of components representing the constraint region, the parameterization of those components, and the rate of algorithm update. Greater accuracy in the *a posteriori* probability estimate may be achieved by using multiple components for the target density itself (reference 8) or by representing it by a multivariate Gram-Charlier series or Edgeworth expansion (references 9-11). The efficacy of these representations will be the subject of future investigations.

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